Hydrogen Bond Studies

119. A Deuteron Magnetic Resonance Study of Sodium Perchlorate Monohydrate, NaClO₄ \cdot D₂O, at 25 $^{\circ}$ C and -55 $^{\circ}$ C

Bo Berglund and Jörgen Tegenfeldt

Institute of Chemistry, University of Uppsala, Box 531, S-751 21 Uppsala, Sweden

(Z. Naturforsch. 32 a, 134-139 [1977]; received December 14, 1976)

The quadrupole splittings in the deuterium NMR spectrum from single crystals of deuterated sodium perchlorate monohydrate, NaClO₄·D₂O, have been measured at 25 °C and -55 °C. At room temperature, the 180° flip frequency of the D₂O molecules is large compared to the difference between the splittings for the two deuterons. The average quadrupole coupling constant ($e^2 \ q \ Q/h$) and asymmetry parameter η are 134.1(4) kHz and 0.621(5), respectively. At -55 °C, the electric field gradient tensors for each of the two deuterons were observed corresponding to slowly flipping water molecules, and $e^2 \ q \ Q/h$ and η for the two deuterons are 231.5(6) and 226.7(6) kHz and 0.196(4) and 0.195(5), respectively. The results indicate that the hydrogen positions in NaClO₄·H₂O are dynamically disordered.

Introduction

An infrared spectroscopic study of sodium perchlorate monohydrate, NaClO4·H2O, revealed an unusually high (3550 cm⁻¹) O-H stretching frequency for the water molecule 1. This shows that the water molecule is involved in very weak hydrogen bonding. Recently, the crystal structure of NaClO₄. H₂O was determined at room temperature using X-ray diffraction 2. The structure and some aspects of the dynamics of the water molecule in NaClO4. H₂O have also been studied at room temperature using neutron diffraction³. The results of these investigations provide additional support for the idea that the water molecule is very weakly hydrogen bonded. In the neutron diffraction study, very large apparent vibrational amplitudes were observed for the hydrogens in the water molecules; these abnormal amplitudes were discussed in terms of a possible disorder of the hydrogen positions.

A deuteron magnetic resonance study of a solid hydrate reveals the electric field gradient (EFG) tensors at the deuteron sites in the water molecules, and these EFG tensors are very sensitive to the hydrogen bond arrangement. A knowledge of the EFG tensors in $NaClO_4 \cdot D_2O$ would therefore provide valuable information on the hydrogen bonding in the compound, as well as resolve the question of a possible disorder. This work forms part of a series of studies of the EFG tensors at the water deuteron sites in different hydrates in progress at this Institute.

Reprint requests to Doc. J. Tegenfeldt, Institute of Chemistry, University of Uppsala, Box 531, S-751 21 Uppsala, Schweden.

Experimental

Commercially obtained NaClO₄·H₂O was heated to about 120 °C to remove the crystallization water and then dissolved in heavy water. Single crystals of NaClO₄·D₂O were obtained from this solution by slowly lowering the temperature by roughly one degree per day. The degree of deuteration was checked by infrared spectroscopy, and found to be not greater than 80%. Since this result was unexpectedly low, the deuteration procedure was carried out a second time, taking even greater care to exclude normal water. The same result was obtained, however. This phenomenon was also observed in the preparation of deuterated crystals of BeSO₄ · 4 D₂O ⁴. The method normally produces almost completely deuterated single crystals of hydrates 5, but for some reson it works less well for certain hydrates. The isotope effect on the quadrupole coupling constant in HDO molecules compared with D₂O molecules has been measured in potassium oxalate monohydrate 6. The effect was small, however, so that the relatively low degree of deuteration is unimportant for our purposes.

Three crystals of roughly the same size (about $5\times5\times10~\text{mm}^3$) were sealed into thin-walled glass tubes to prevent decomposition. Each crystal was then mounted on a goniometer head and the crystal orientation determined on a 4-circle X-ray diffractometer. The goniometer head was transferred to the NMR spectrometer without disturbing the crystal orientation and mounted on a device which allows a precise rotation through an angle Φ (reproducibility $\approx 0.1^{\circ}$) about an axis perpendicular to the magnetic field. This procedure for the determination of the crystal rotation axis has proved to be very accurate 4,5 . The rotation axis for each of the crys-



Dieses Werk wurde im Jahr 2013 vom Verlag Zeitschrift für Naturforschung in Zusammenarbeit mit der Max-Planck-Gesellschaft zur Förderung der Wissenschaften e.V. digitalisiert und unter folgender Lizenz veröffentlicht: Creative Commons Namensnennung-Keine Bearbeitung 3.0 Deutschland Lizenz.

This work has been digitalized and published in 2013 by Verlag Zeitschrift für Naturforschung in cooperation with the Max Planck Society for the Advancement of Science under a Creative Commons Attribution-NoDerivs 3.0 Germany License.

Zum 01.01.2015 ist eine Anpassung der Lizenzbedingungen (Entfall der Creative Commons Lizenzbedingung "Keine Bearbeitung") beabsichtigt, um eine Nachnutzung auch im Rahmen zukünftiger wissenschaftlicher Nutzungsformen zu ermöglichen.

On 01.01.2015 it is planned to change the License Conditions (the removal of the Creative Commons License condition "no derivative works"). This is to allow reuse in the area of future scientific usage.

Table 1. Rotation axes for the three crystals. The vector components are given in the coordinate system defined by the unit cell axes.

Crystal No		Rotation axis				
1 2 3	$0.05714 \\ -0.02336 \\ 0.06441$	-0.06305 0.16854 0.00407	-0.01084 -0.00138 0.00034			

tals is given in Table 1. Two sets of deuteron spectra were collected for each of the three crystals at 7.0 MHz by rotation over a Φ range of 180° in steps of 10°: one set at 25 °C and one set at -55 °C.

The audio frequency modulation amplitude was about 0.5 mT and the r.f. field was held below saturation level. To increase the signal-to-noise ratio, about 100 spectra per crystal orientation were accumulated in an IBM-1800 computer. The low temperature was obtained by blowing a cold nitrogen gas-stream on the crystals. The magnetic field was swept with a Fieldial unit at rates varying between 5 and 10 mT/min.

The magnetic field sweep was calibrated by recording the positions of the deuteron signal from a water sample at various frequencies measured with a frequency counter. The reproducibility of the sweep rate was better then 0.1% over the whole period of data collection. The linearity of the part of the sweep used was better than 0.3% of the total sweep width.

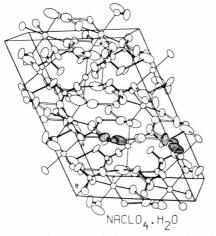
Results

The diffraction studies on $NaClO_4 \cdot H_2O^{2,3}$ have demonstrated that the structure, shown in Fig. 1, belongs to the monoclinic space group C2/c, (a =

15.5422(13), b=5.5399(5), c=11.0455(11), $\beta=110.666(8)$, Z=8). There are eight symmetry related water molecules in the unit cell, forming two groups of four molecules, and all the molecules within each group producing identical spectra. A DMR-spectrum will therefore contain four doublets if the water molecules undergo no flipping motion or flip at a sufficiently low rate. On the basis of signal-to-noise considerations, $-55\,^{\circ}\mathrm{C}$ was found to be the most convenient temperature for studying the EFG tensors of the "static" water molecule. As the number of splittings was small, no difficulties arose in assigning a given splitting to the correct deuteron and symmetry operation.

The method for calculating the EFG tensors from the observed quadrupole splittings has been described briefly in Reference 5. The eigenvalues and eigenvectors of the crystallographically independent EFG tensors were obtained in a least-squares analysis of the observed splittings calculated on an IBM 370/155 computer using the program QSPL3 7. The analysis of the data from 25 °C was based on 98 observed splittings and the r. m. s. deviation between observed and calculated splittings (from the final EFG tensors) was 0.18 mT. Corresponding quantities at -55 °C were 179 observations and 0.22 mT, respectively. At each temperature, all data from all three crystals were used simultaneously in the analysis, including splittings from symmetry related EFG tensors.

The calculated eigenvalues and eigenvectors at $-55\,^{\circ}\text{C}$ and $25\,^{\circ}\text{C}$ are given in Table 2 and Table 3, respectively. A comparison between crystal-



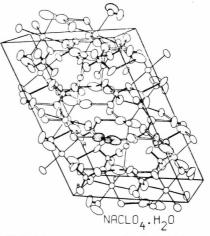


Fig. 1. A stereoscopic illustration of the structure of NaClO₄·H₂O. The two water molecules which are magnetically non-equivalent with respect to the external magnetic field are shaded. The figure is based on parameters from Reference ³.

Table 2. Eigenvalues and eigenvectors of the EFG tensor for the two deuterons at $-55\,^{\circ}\text{C}$. The vector components are given in the coordinate system defined by the unit cell axes.

	Eigenvalues (kHz)		Eigenvectors	
D_1	-93.1 (6) * -138.4 (7) 231.5 (6)	0.0103 (8) 0.0649 (1) -0.0203 (1)	$-0.1783(4) \\ 0.0279(24) \\ -0.0013(3)$	0.0086 (9) 0.0587 (1) -0.0764 (1)
D_2	-91.2(7) $-135.5(7)$ $226.7(6)$	0.0149 (7) 0.0643 (1) 0.0192 (1)	$\begin{array}{c} 0.0518(4) \\ 0.0383(7) \\ -0.1686(1) \end{array}$	-0.0771(6) $0.0575(8)$ $-0.0106(1)$

^{*} Numbers in paranthesis here and in the following are estimated standard deviations in the least significant digits.

lographic data and the DMR data at $-55\,^{\circ}\mathrm{C}$ is given in Table 4. It should be noted that the DMR data used in this comparison were recorded at $-55\,^{\circ}\mathrm{C}$ whereas the neutron diffraction study was carried out at $25\,^{\circ}\mathrm{C}$.

Discussion

The EFG tensors

Several DMR studies on deuterated hydrates have shown that the deuterium quadrupole coupling constants are correlated to the hydrogen-bond distance: the shorter the hydrogen bond the smaller the value of $e^2 q Q/h$. Furthermore, the asymmetry parameter

 η is normally close to 0.1 for the EFG tensor of a "static" water molecule. Some examples are given in Table 5. As the average O...O hydrogen-bond distances in NaClO₄·H₂O are larger than 3.0 Å (see Table 4) for both hydrogens, the quadrupole coupling constants for the two deuterons in NaClO₄. D₂O are expected to be around 250 kHz. Our observed results (231.5(6)) and 226.7(6)) are considerably smaller. Furthermore, the asymmetry parameters η are 0.196(4) and 0.195(5), or about twice their normal values. At 25 °C the observed EFG tensors for the two deuterons are indistinguishable because of the rapid flip motion of the water molecule, and are closely equal to the average of the two EFG tensors observed at -55 °C. The quadrupole coupling constant, 134.1(4) kHz, is unexpectedly high, and the asymmetry parameter, 0.621(5), differs from the normally observed value, 0.8 - 0.9. It would therefore appear that whatever causes the abnormal EFG tensor at −55 °C is also operative at 25 °C. There are several possible explanations for the origin of this abnormal behaviour.

1) A strong interaction between the water molecule and the metal cation bonded to the water oxygen can have a considerable effect on the EFG tensors ⁴. This effect is likely to be unimportant in the present case, as the $Na^+ \dots O(W)$ distances in $NaClO_4 \cdot H_2O$ are comparable to those in $NaHC_2O_4 \cdot$

Table 3. Eigenvalues and eigenvectors of the averaged EFG tensor at 25 °C together with the mean value of the two tensors in Table 2. The components are given in the coordinate system defined by the unit cell axes.

	Eigenvalues (kHz)		Eigenvectors	
Exp. data at 25 °C	$-25.4(3) \\ -108.7(4) \\ 134.1(4)$	0.0006(1) $-0.0250(1)$ $0.0640(1)$	0.1384 (4) 0.1083 (5) 0.0411 (3)	$-0.0578(3) \\ 0.0521(2) \\ 0.0575(1)$
Mean value of the EFG tensors at -55 $^{\circ}$ C	$-22.1 \\ -114.8 \\ 136.9$	$ \begin{array}{r} 0.0009 \\ -0.0244 \\ 0.0643 \end{array} $	$0.1444 \\ 0.1019 \\ 0.0367$	-0.0539 0.0554 0.0582

Table 4. Some structural quantities related to the EFG tensor at the deuteron nuclei. α_1 , α_2 and α_3 are the angles between the z-principal axis and the O-H, H...O and O...O vectors. β is the angle between the y-principal axis and the normal to the water molecule plane. The crystallographic data are taken from Reference ³.

Deuteron	Hydrogen- bond angle O-HO(°)	О-Н (Å)	HO (Å)	OO (Å)	$\frac{e^2qQ/h}{({\rm kHz})}$	η	β (°)	α ₁ ([◦])	a_2 (°)	α ₃ (°)
D_1	140.4 (9) 135.2 (9)	0.905(6)	2.389 (7) 2.378 (10)	3.141 (3) 3.087 (3)	231.5(6)	0.196(4)	3.5(1.1)	2.1(7)	41.5 (2) 42.0 (2)	31.0(1) 30.9(1)
D ₂	155.5 (7) 122.0 (7)	0.935(7)	2.152 (8) 2.733 (7)	3.028(3) 3.325(3)	226.7(6)	0.195(5)	0.2(1.0)	1.0(5)	25.2 (2) 57.1 (2)	17.9(1) 43.3(1)

Table 5. Some representative quadrupole coupling constants
and asymmetry parameters of EFG tensors for deuterium in
hydrates. The first reference relates to the distance and the
second to the NMR data.

Compound	O O (Å)	e^2qQ/h	η	Ref.
$NaDC_2O_4 \cdot D_2O$	2.808 2.826	228.5 235.2	0.127 0.090	8, 9
$\mathrm{LiHCOO} \cdot \mathrm{D_2O}$	2.714 2.896	$198.7 \\ 231.3$	$0.060 \\ 0.097$	10, 5
$\mathrm{Li}_2\mathrm{SO}_4\cdot\mathrm{D}_2\mathrm{O}$	2.87 2.95	236.7 239.8	$0.091 \\ 0.126$	11, 4
$Ba(ClO_3)_2 \cdot D_2O$	2.891	243.5	0.074	12, 13
$Na_2S_2O_6 \cdot 2D_2O$	2.85 2.98	$\frac{236}{244}$	$0.116 \\ 0.103$	14, 15
${\rm K_2C_2O_4 \cdot D_2O}$	2.754	209.7	0.090	16, 17

H₂O for which normal EFG tensors were found in a DMR study 9.

2) It has been suggested that the asymmetry parameter of the EFG tensor of a deuteron will increase as a result of participation in a non-linear hydrogen bond ^{18, 19}. In NaClO₄·D₂O, each O-H group points towards two hydrogen bond acceptors (a bifurcated bond, cf. Fig. 2) which are not colinear

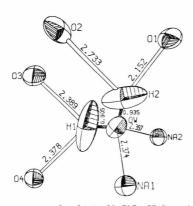


Fig. 2. The water molecule in NaClO₄·H₉O and its immediate environment. The figure is taken from Reference 3.

with the O-H direction. Both will therefore add to the non-axiality of the EFG tensors. This effect cannot be excluded from being at least partially responsible for the high η in the present case. It is difficult to make a quantitative estimate of the importance of this effect, however.

3) The most probable major contribution to the non-axiality is related to the very large apparent vibrational amplitudes for the hydrogens observed in the neutron diffraction study. Independent of whether these apparent vibrational amplitudes are

due to actual vibrations or to a dynamical disorder, they will strongly affect the observed EFG tensors. Furthermore, by far the largest amplitude was in the plane of the water molecule. A librational motion in this plane, i.e. about the eigenvector y of the equilibrium EFG tensor, will not affect the eigenvalue $|V_{yy}|$ but will reduce the observed eigenvalues $|V_{zz}|$ and $|V_{xx}|$. The average of $\eta = (|V_{yy}| - |V_{xx}|)/|V_{yy}|$ $|V_{zz}|$ will therefore increase as a result of such a motion, as is actually observed. An estimate of the effect of a libration which is predominantly in the plane of the water molecule may be obtained as follows. If θ_x , θ_y and θ_z represent rotational displacement coordinates about the equilibrium eigenvectors x, y and z, then the relation z^{20} between the average EFG components $\langle V_{ii} \rangle$, the equilibrium values V_{zz}^0 and η^0 and the mean square amplitudes of libration gives that

$$\frac{\langle V_{xx} \rangle - \langle V_{yy} \rangle}{\langle V_{zz} \rangle} = \frac{V_{zz}^{0}}{\langle V_{zz} \rangle}$$

$$\cdot \left\{ \eta_{0} \left[1 - \frac{1}{2} (\langle \theta_{x}^{2} \rangle + \langle \theta_{y}^{2} \rangle) - 2 \langle \theta_{z}^{2} \rangle \right] \right.$$

$$+ \frac{3}{2} \left(\langle \theta_{y}^{2} \rangle - \langle \theta_{x}^{2} \rangle \right) \right\}.$$
(1)

Note that, strictly speaking, the left-hand side of this relation is not in general the asymmetry parameter $\langle \eta \rangle$ of the average EFG tensor. This is because the eigenvectors of the average EFG tensor will not in general coincide with those of the equilibrium EFG tensor. Therefore, since $\langle V_{xx} \rangle$, $\langle V_{yy} \rangle$ and $\langle V_{zz} \rangle$ are components of the average EFG tensor referred to the equilibrium eigenvectors x, y and z, these averaged quantities will not be eigenvalues of the average EFG tensor and they will not define $\langle \eta \rangle$. However, for the special case where the librational modes correspond to rotations about axes parallel to x, y and z, the eigenvectors for the average EFG tensor will coincide with x, y and z, and we can write $\langle \eta \rangle = (\langle V_{xx} \rangle - \langle V_{yy} \rangle) / \langle V_{zz} \rangle$. In our case, the major motion is about an axis normal to the plane of the water molecule, i. e. roughly about y. The averaged EFG eigenvectors will therefore probably not differ much from the equilibrium values, and we may take Eq. (1) as an estimate of $\langle \eta \rangle$.

$$egin{align*} \langle \eta
angle &pprox rac{V_{zz}^0}{\langle V_{zz}
angle} \left\{ \eta_0 - rac{1}{2} \langle heta_x^2
angle \left(3 + \eta_0
ight) \ &+ rac{1}{2} \langle heta_y^2
angle \left(3 - \eta_0
ight) - 2 \ \eta_0 \langle heta_z^2
angle
ight\}. \ \end{aligned}$$

Since
$$\eta_0 \approx 0.1$$

$$\langle \eta \rangle \approx \frac{V_{zz}^0}{\langle V_{zz} \rangle} \left\{ \eta_0 + \frac{3}{2} (\langle \theta_y^2 \rangle - \langle \theta_x^2 \rangle) - 2 \eta_0 \langle \theta_z^2 \rangle \right\}.$$

Since $\langle \theta_y^2 \rangle \gg \langle \theta_x^2 \rangle$, $\langle \theta_z^2 \rangle$ and $\eta_0 \approx 0.1$, the most important correction term in the bracket is $\frac{3}{2}(\langle \theta_y^2 \rangle - \langle \theta_x^2 \rangle)$. Neglecting the term $2 \eta_0 \langle \theta_z^2 \rangle$, taking 250 kHz as a lower limit for V_{zz}^0 and taking $\eta_0 = 0.1$, a rough estimate of the $(\langle \theta_y^2 \rangle - \langle \theta_x^2 \rangle)$ value required to give the observed $\langle \eta \rangle = 0.2$ would be $0.06 \, \mathrm{rad}^2$. This is considerably smaller than the amplitudes observed in the neutron diffraction study. We thus conclude that the in-plane motion of the water molecule is sufficiently large to account for the abnormal asymmetry parameter, at least at room temperature.

The possibility of Disorder

The neutron diffraction study of NaClO₄·H₂O³ showed a remarkably large r.m.s. amplitude of displacement of the hydrogens in the plane of the water molecule, as shown in Figure 2. This result can either be due to disorder in the hydrogen positions or to an extremely large librational motion in the water molecule plane (a large rocking mode). In the first case, the water molecule will have at least two possible equilibrium orientations, related by a rotation about an axis roughly normal to the plane of the water molecule. In the second case, however, there will be just one equilibrium orientation. Furthermore, if disorder is the predominant cause of the large in-plane amplitude, it is most probably dynamical, i.e. the water molecule performs random jumps between the possible equilibrium positions. For it to be a static disorder, additional splittings should be observed, at least at certain crystal orientations.

The measured EFG tensor at 25 °C is compared in Table 3 with the EFG tensor obtained as the mean value of the two EFG tensors measured at -55 °C. It is clear that these two tensors are very similar. The temperature dependence of the EFG tensor is thus rather small. This result is not compatible with the suggestion that the very large orientational freedom of the water molecule observed with neutron diffraction is due to a rocking vibration. Such a large librational amplitude would correspond to a very low vibrational frequency, and therefore to a large temperature dependence both of the librational amplitude and the vibrationally averaged EFG tensor. On the other hand, if the hydrogens are disordered, the temperature dependence of the vibrational amplitude is expected to be small; the major effect of a temperature change would be to alter the jump rate between the possible equilibrium orientations of the water molecule. Our data therefore favour an interpretation in terms of dynamical disorder rather than large vibrational amplitude.

It should also be noted that, in the neutron diffraction study, a crude estimate of the librational frequency corresponding to the observed amplitudes gave the value 146 cm⁻¹. This is considerably below any observed rotational vibration for a water molecule. Taken together with our results, this provides additional evidence for the disorder in spite of the many approximations involved in the vibrational analysis.

In the IR study of $NaClO_4 \cdot H_2O^{\,1}$ very narrow O-D stretching bands were observed down to $-165\,^{\circ}C$. This indicates either that the water molecule is ordered or, if disorder exists, that the disorder is between orientations of the water molecule which produce very closely similar O-D stretching frequencies. The observed IR spectra are thus compatible both with a disordered structure and with a large in-plane vibration. In our view, however, the total experimental evidence favours a disordered structure.

It is rather unfortunate that this apparent disorder exists in $NaClO_4 \cdot H_2O$ since it would be very interesting to know the EFG tensors at the deuteron sites in this very weakly hydrogen-bonded water molecule. It is probably impossible to resolve the EFG tensors for the disordered deuterons even if the temperature is decreased further, since the jumps between two adjacent positions are likely to be hindered by a very low activation energy, and would therefore be very difficult to "freeze-in". However, if the various equilibrium hydrogen positions can be resolved by low-temperature neutron diffraction, it will be possible (with some approximations) to calculate $e^2 \, q \, Q/h$ for each of the equilibrium orientations of the water molecules in $NaClO_4 \cdot D_2O$.

Acknowledgements

The authors wish to thank Prof. Ivar Olovsson for the facilities he has placed at their disposal. We also thank Drs. A. Eriksson and J. Lindgren for valuable discussions and comments, and Mr. H. Karlsson for invaluable assistance on the X-ray diffractometer. This work has been supported by grants from the Swedish Natural Science Research Council which are hereby gratefully acknowledged.

- ¹ G. Brink and M. Falk, Can. J. Chem. 48, 2096 [1970].
- ² B. Berglund, J. O. Thomas, and R. Tellgren, Acta Cryst. **B 31,** 1842 [1975].
- ³ B. Berglund, R. Tellgren, and J. O. Thomas, Acta Cryst.
- B 32, 2444 [1976].B. Berglund and J. Tegenfeldt, Submitted for publication.
- ⁵ B. Berglund, J. Lindgren, and J. Tegenfeldt, J. Mol. Structure 21, 135 [1974].
- ⁶ B. Pedersen and K. Håland, Chem. Phys. Letters 2, 61 [1968].
- ⁷ J. Tegenfeldt, UUIC-B13-6. Institute of Chemistry, University of Uppsala, Sweden 1973.
- ⁸ R. Tellgren and I. Olovsson, J. Chem. Phys. 54, 127 [1971].
- ⁹ B. Berglund and J. Tegenfeldt, Mol. Phys. 26, 633 [1973].
- 10 R. Tellgren, P. S. Ramanujam, and R. Liminga, Ferroelectrics 6, 191 [1974].

- ¹¹ H. G. Smith, S. W. Peterson, and H. A. Levy, J. Chem. Phys. 48, 5561 [1968].
- 12 S. K. Sikka, S. N. Momin, H. Rajagopal, and R. Chidambaram, J. Chem. Phys. 48, 1883 [1968].
- ¹³ T. Chiba, J. Chem. Phys. 39, 947 [1963].
- ¹⁴ I. Berthold and A. Weiss, Z. Naturforsch. 22 a, 1440 [1967].
- ¹⁵ S. Ketudat, I. Berthold, and A. Weiss, Z. Naturforsch. 22 a, 1452 [1967].
- ¹⁶ A. Sequeira, S. Srikanta, and R. Chidambaram, Acta. Cryst. B 26, 77 [1970].
- ¹⁷ J. W. McGrath and G. W. Ossman, J. Chem. Phys. 46, 1824 [1967].
- ¹⁸ T. Chiba, J. Chem. Phys. **41**, 1352 [1964].
- 19 J. O. Clifford, J. A. S. Smith, and F. P. Temme, J. Chem. Soc. Faraday Trans. II, 71, 1352 [1975].
- ²⁰ R. Sjöblom and J. Tegenfeldt, Submitted for publication.